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Trailing-Edge Noise from Hovering Rotors

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A method has been developed to predict the high-frequency broadband noise due to the interaction of convecting turbulent eddies with the trailing edges of a hovering rotor. The trailing-edge noise from each blade was modeled as moving point dipole noise with spanwise loading corrections. This point dipole approximation was checked by applying the concept to a stationary airfoil in a moving medium with excellent results. In order to estimate the strength of the point dipole, the trailing-edge noise theory of Amiet was used. The method was applied specifically to blade boundary-layer turbulence and compared to incident atmospheric turbulence noise. The results indicate that the relative importance of these two mechanisms is related to the intensities and the magnitudes of of the length scales of the inflow and boundary-layer turbulence. Trailing-edge noise is shown to be important in low inflow turbulence conditions. The approach which was developed is also applicable to other blade-turbulence interaction mechanisms such as local stall and tip noise.

Introduction

BROADBAND rotor noise becomes dominant in the absence of impulsive noise as, for example, in hovering helicopter flight. Broadband noise is significant particularly for large, low-speed rotors and in the frequency range of 150-10,000 Hz. The broadband component of the rotor noise spectrum is attributed to random loads on the blades. A number of phenomena have been proposed which might contribute to such loading fluctuations.[1] One significant source of loading fluctuation is the passage of the blades through turbulent velocity fluctuations in the rotor plane.[2] This inflow turbulence may come from the atmosphere or from the wakes of preceding blades. Several other mechanisms also exist for random loadings due to the flow around a single blade. If the blade boundary layers are largely laminar, certain tones are radiated which can be related to instability of the boundary layer or laminar wake.[3,4] However, this effect does not occur on most full-scale rotors when the Reynolds number is high enough to produce turbulent boundary layers. Direct radiation from turbulent boundary layers on the blade surfaces is quite weak,[5] but when turbulent eddies pass over the trailing edge of a blade, somewhat more sound is radiated.[6] Other sources of broadband noise from rotor blades can be due to turbulence in locally stalled regions[7] or due to tip flow effects.[8-11] The mechanism of acoustic radiation due to local stall is the interaction of stall-generated turbulent eddies with the airfoil trailing edge. The importance of turbulence in blade tip flow has been analyzed and it is becoming clear that the interaction of turbulent eddies with the trailing edge near the blade tip can be an important noise generation mechanism. In the absence of excessive inflow turbulence, trailing-edge noise may become a dominant source of broadband rotor noise at least under certain operating conditions. Understanding of the details of this noise generation mechanism is still in a state of flux both experimentally and theoretically even for a stationary airfoil. Since the problem of trailing-edge noise at low Mach numbers was first examined by Powell,[5] various subsequent analyses have considered very simplified prototype configurations in which the stationary wing and trailing edge are modeled by a semi-infinite rigid plate with prescribed nearby flows or acoustic sources. Unfortunately, many of these analyses have been based on different flowfield approximations and different surface/observer geometries. Therefore, a direct comparison of the models was difficult. Recently, Howe[12] undertook a comparison of various half-plane models for a situation of consistent flowfield conditions. He concluded that the predominant source mechanism is dipole in nature and that when cast in terms of a common system of flow parameters for a semi-infinite plane, most of the theories predicted the same results for low Mach number conditions. However, only a few investigators have included the effects of rotation in their models thus far. Munch et al.[13] attempted to predict the rotor broadband noise level due to local stall near the blade tip by applying the trailing-edge noise theory of Ffowcs Williams and Hall.[14] Their method includes the scaling laws and requires experimentally measured sound radiation information on the rotating axis. Their results overpredicted the noise level by 5-8 dB. More recently Fink[15] developed a semi-empirical formula for the prediction of on-axis rotor trailing-edge noise due to turbulent boundary layers based on the same model. Though such semi-empirical expressions can be quite simple to apply and accurate for those rotors and flow conditions considered in the sample used to define them, their validity under other circumstances is open to doubt.

The overall objectives of the present study were to provide a quantitative understanding of the contribution of trailing-edge noise to broadband rotor noise spectra and directivity as a function of relevant variables and use these results to assess the relative importance of this noise mechanism compared to others. It was desired to develop a method which could treat local stall and tip flows as well as the turbulent boundary-layer case. An additional objective was to remove as much empiricism as possible by developing a relatively simple, but first principles based model. In the present analysis, the acoustic spectrum and directivity of the noise is related to the blade surface pressure spectrum produced by the convecting turbulent flow. There are no adjustable constants in the theory. In the present work we modeled a trailing-edge noise source on a rotating blade by a rotating point dipole, though the trailing-edge noise source has somewhat different characteristics. This will be shown to be a good approximation for angles not close to the rotor plane. The strength of this point dipole was obtained from Amiet's model[16,17] of trailing-edge noise radiation from a fixed blade. His model serves our purpose well because it is complete in

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itself and does not need additional modeling or empiricism but only experimentally determined surface pressure spectra far from the edge. The present study gives a specific formula for far-field sound radiation due to the interaction of turbulent boundary layers with the trailing edges of the rotating blades. This was the only case for which the convected pressure characteristics had been sufficiently well-documented for direct application. The present work shows that trailing-edge noise due to turbulent boundary layers can be more important than the atmospheric inflow turbulence noise in the high-frequency ranges under certain operating conditions of a rotor.

Acoustic Formulation

The basic geometry for the analysis is shown in Fig. 1. A subsonic rotor rotates in the x-y plane at angular frequency Ω. Here we are concerned with the sound radiation due to the convection of turbulent flows over the trailing edges of rotating blades. Although many different trailing-edge noise theories have been developed, they differ on items such as whether to apply the Kutta condition and its importance and on the locations, convection speeds, and types of multipole sources. In particular, the question of application of the Kutta condition at the trailing edge is unresolved with substantial differences occurring in the predicted levels depending on the presence or absence of this condition. Furthermore, for finite chord surfaces, the relationship is expected to become more complicated; indeed, the trailing-edge noise problem has not been fully described analytically for finite surface geometries. However, it was felt that Amiet's theory[16,17] of trailing-edge noise would be most suitable for the present work, because the theory is relatively complete in itself and convenient for calculating the induced surface pressure distribution due to edge effects. Before applying this theory to rotor trailing-edge noise prediction, we assessed the theory by comparing with the existing experimental results as shown in Fig. 2. The predicted spectrum level of trailing-edge noise due to turbulent boundary layer lies below the measured spectra. This discrepancy may result from the imposition of the Kutta condition at trailing edge or from the use of a flat-plate boundary-layer pressure spectrum as discussed later.

As the application of the theory for a stationary surface to rotating systems involves accounting for radial variation of mean and unsteady flow parameters, the departures from quasi-two-dimensional flow situations at the tip of a rotating blade and rotating acoustic source effects, some simplifications had to be made to make the problem tractable. Trailing-edge noise is somewhat different from point dipole noise in several aspects such as the noncompactness effects, velocity dependence, and the effects of moving medium and sources. However, as its noise mechanism is basically dipole in nature and since the analysis we will use requires a statistically stationary force component in the observer direction, we model a rotating blade by a rotating point dipole to simplify the problem. The point load approximation made here is exactly applicable only in the limit of acoustic wavelength large compared to chord and rotor span. However, the characteristic dimensions of some of the boundary layer or other turbulence on the blade are small compared to the scale of the blade, therefore high frequencies are generally associated with trailing-edge noise. We made theoretical corrections to account for these differences which improved the predicted results. The spanwise loading distribution due to turbulent flow is correlated over distance of the order of the inverse wavenumber of the particular convecting turbulence component being considered and this effect easily was accounted for by assuming random phase between radiated sound from uncorrelated parts of the span. On the other hand, chordwise loading effects influence both the airfoil unsteady aerodynamic response and the acoustic radiation directivity at high frequencies. Thus, the trailing-edge noise has approximately a $\cos\theta/2$ directivity pattern compared to $\sin\theta$ for point dipole. However, the present knowledge of finite thickness and span noncompactness effects is still only qualitative. Hence, it is very difficult to include such effects in the prediction of rotor trailing-edge noise. Fortunately, the directivity of the radiation from rotating blades is already averaged out for a fixed observer location. The overall effect on rotor radiation can still be reasonably approximated by dipole directivity even for trailing-edge noise, for angles that are not too close to the rotor plane as shown subsequently. Thus, in the present work, we adopted the dipole model.

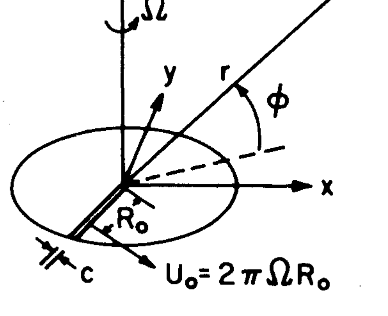

Fig. 1 Rotor geometry showing definitions of symbols.

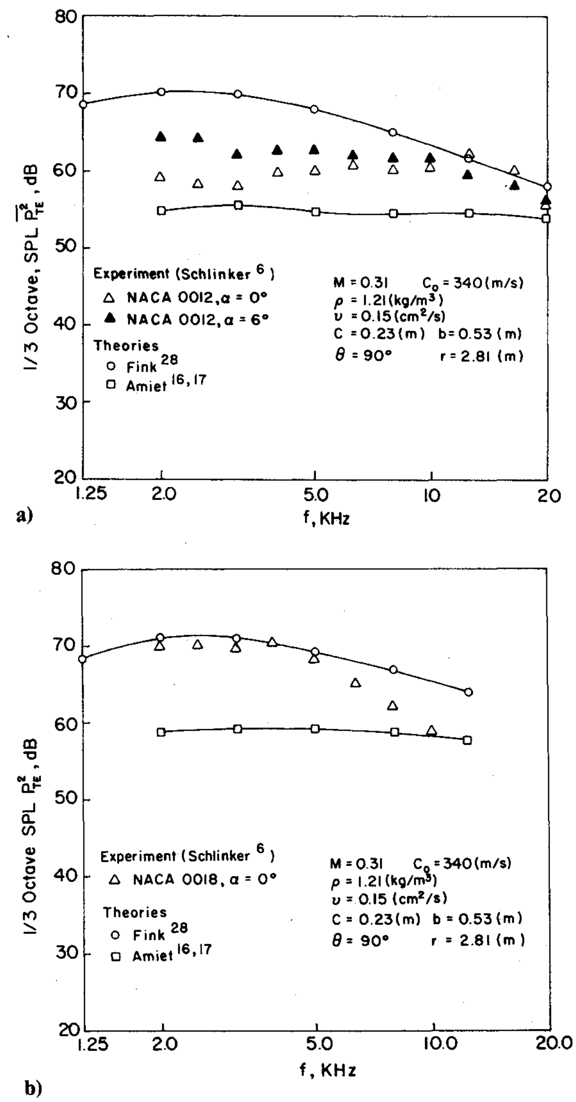

Fig. 2 Comparison of the trailing-edge noise theories with experiments, a) NACA 0012 airfoil, and b) NACA 0018 airfoil.

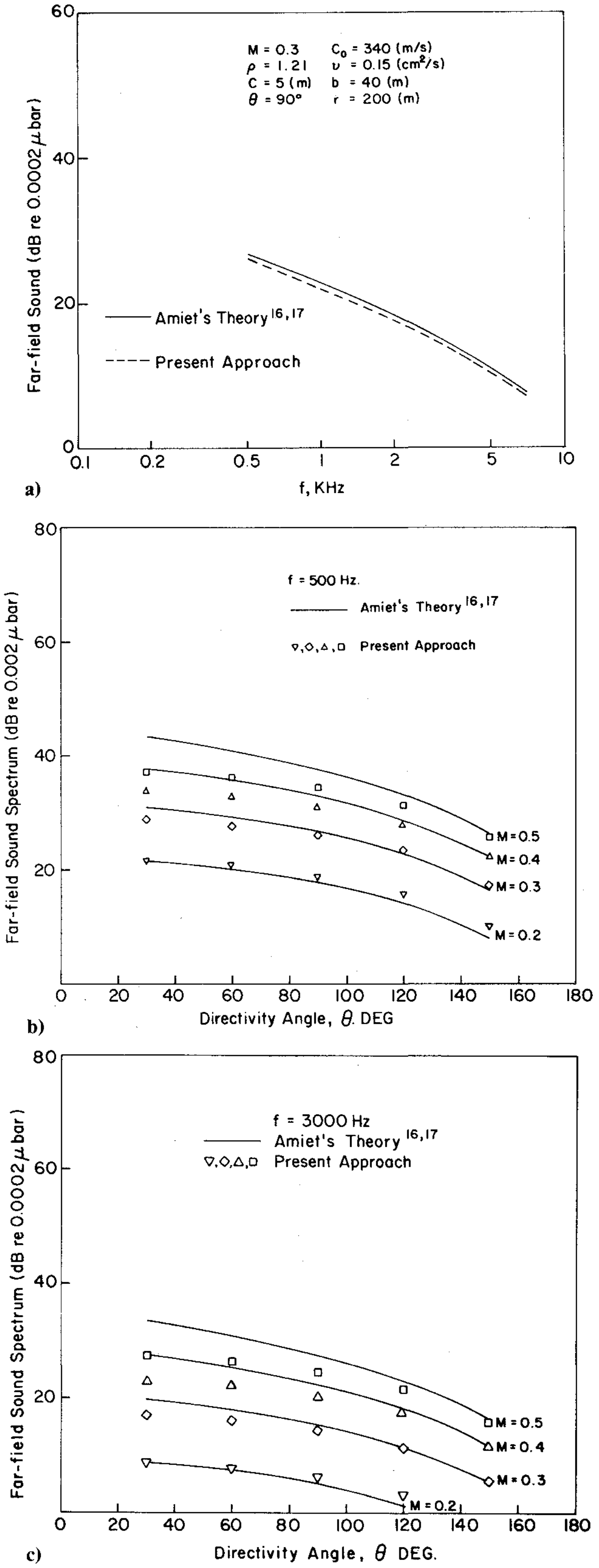

Fig. 3 a) Comparison of Amiet's theory with the present point dipole approach: at overhead position, b) directivity comparisons: $f=500$ Hz, and c) directivity comparisons: $f=3000$ Hz.

In order to obtain better agreement with the experimental results near the rotating plane, we should also include the contributions from other possible sources such as the torque component of a fluctuating lift and drag force and unsteady thickness quadrupole sources at lower Mach numbers as discussed by Hawkings.[18] Also, as the Mach number approaches 1, we should include the effects of blade thickness and other quadrupole sources near the rotor plane. However, these additional noise sources are not well enough understood to be added to the present prediction formula of rotor trailing-edge noise. Thus, in the present work we are concerned mainly with the prediction of rotor trailing-edge noise for angles that are not too close to the rotor plane.

We tested this point dipole approximation to trailing-edge noise sources by applying the concept to a stationary airfoil in a moving medium. The blade was modeled as a stationary point dipole, with a series of corrections to include the spanwise load variations, velocity dependence change, and static directivity of trailing-edge noise. The effect of the convecting medium was not included in the calculation. However, the approximation predicted the spectrum level at the directly overhead observation position within an error of about 1 dB or less compared with Amiet's results as shown in Fig. 3. Also as shown in Fig. 3 the results are generally good in a range of angles from 30 to 150 deg. Thus, the dipole approximation with proper correction is applied to the case of a rotor in the present work. The fact that the blades are moving can be looked at in terms of Doppler shifts in frequency. However, here we take the equivalent point of view that the variations of the pressure distribution associated with the presence or absence of the blade at various spatial locations become time variations as seen in observer coordinates. Thus, a range of frequencies are generated at blade passing harmonics $nB\Omega$ where n is the harmonic number, B the number of blades, and Ω the shaft frequency.

Another rotor effect which does not occur for a stationary blade can be due to the correlation of loadings between blades. However, if the blades are independent of other blades' wakes there is no reason to expect any correlation between turbulent boundary-layer loadings on separate blades. As a result, the radiation from separate blades will be uncorrelated. Thus, for broadband loadings we are simply to add the uncorrelated sound power spectral densities of the radiation field from each blade. The radiated spectral density of a single blade, considered as a rotating point load which is statistically stationary in the observer's direction, can be expressed as shown by Ffowcs Williams and Hawkings[19] as

$$\langle S(x,f)\rangle = \frac{f^2}{4\rho c_0^3 r^2}\sum_{n=-\infty}^{n=\infty} D_r(f-n\Omega)\,J_n^2(f\alpha/\Omega) \tag{1}$$

where D_r is the power spectral density of the force fluctuation in the ϕ direction, f the frequency in Hz, $\alpha = M_0\cos\phi$, c_0 the sound speed, ρ the density of acoustic medium, and r the distance between the observer and the rotor. Since the radiated pressure from a fluctuating force is proportional to $\partial/\partial t$ (force), which for a single frequency component is just $2\pi i f$(force), we see that the intensity spectrum contains the factor f^2. However, any given component of a load spectrum radiates over a range of Doppler shifted frequencies. Conversely, the acoustic spectrum at f involves load fluctuations at frequencies shifted from f by a range of harmonics $n\Omega$. The amplitudes of the Doppler shifted contributions are seen to be proportional to $J_n^2[f\alpha/\Omega]$. The value of $J[f\alpha/\Omega]$ becomes small for $n \gtrsim [f\alpha/\Omega]$. Thus, the primary Doppler effects are shifts $\Delta f = n\Omega$ of less than approximately αf, or using $\alpha = M_0\cos\phi$ we obtain that the frequency shifts satisfy $\Delta f/f \lesssim M_0\cos\phi$ as would be expected from simple Doppler shift ideas.

Blade Loading due to Turbulent Flow

We next find an equivalent point lift loading on the blade to estimate the trailing-edge noise from a rotating blade. The variation of mean flow velocity along the radius will be neglected by introducing the effective velocity at an effective radius. We will not include the effects of thickness, angle of attack, and camber of the airfoil, although these may have

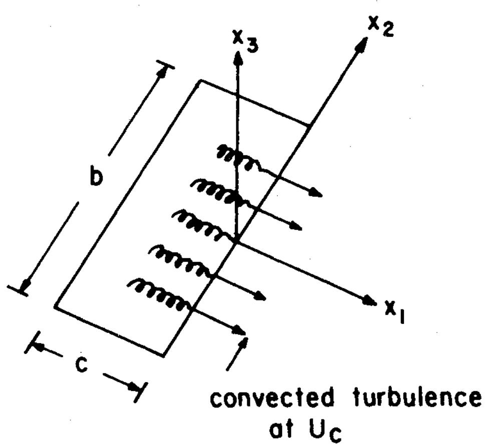

Fig. 4 Basic geometry of blade with convected turbulence.

some small influences on the final result. Therefore, our model of a blade will be a thin, flat plate airfoil at zero angle of attack which makes the mathematical problem simpler. We adopted the same assumption as Amiet that the turbulent velocity field convecting past a trailing edge is unaffected by the presence of the trailing edge; i.e., the turbulence is stationary in the statistical sense as it moves past the trailing edge.[16] This assumption allows the edge noise to be calculated from the spectral characteristics of wall pressure which would exist in the absence of the trailing edge. Recent experiments indicate that the measured trailing-edge pressure spectra (with the exception of an apparent peak which may be related to discrete flow-shedding phenomenon at the trailing edge) correspond to those that had been measured for attached turbulent boundary layers with equal physical properties away from edges.[20,21] Hence, the presence of a boundary discontinuity seems not to affect the properties of the approaching boundary-layer flow. However, it is not clear yet how accurate this assumption will be for other cases, such as separated turbulent flow.

Thus, our model for calculating blade loading consists of a turbulent flow, with stationary statistical properties, convecting past a trailing edge as shown in Fig. 4. Upstream of the edge, this produces a convecting pressure pattern on the airfoil surface. Near the trailing edge, in addition to the convecting pressure pattern, a pressure field of comparable magnitude is induced by the flow adapting to the end of the surface. In the present work, the view is taken that the boundary-layer noise is generated primarily by the surface pressure dipoles near the trailing edge and equal (but anticorrelated) radiation into the upper and lower region is predicted. On the airfoil (assumed to lie in the $x_3=0$ plane) with convecting turbulent fluid passing over the trailing edge with a mean flow U_c in the x_1 direction, a particular spectral component of the convecting surface pressure field with spanwise wavenumber k_2 can be written as

$$P_c(x_1,x_2,t)=P_0(k_1,k_2)\exp\{i[\omega(t-x_1/U_c)-k_2x_2]\} \tag{2}$$

It is assumed first that the surface pressure is uniform along the spanwise direction, i.e., $k_2=0$. The effects of the spanwise variation of surface pressure will be considered later. The induced surface pressure on the $x_3=0$ plane obtained by Amiet[16,17] using the Schwartzschild method can be represented in the following form (for the details of derivations in the present work see Ref. 22):

$$P_I(x_1,0,t)=P_0(K_1)[\mathrm{erf}(-2x_1q/c)^{1/2}-1]$$
$$\times\exp[iw(t-x_1/U_c)] \tag{3}$$

where

$$q=i[M\mu+K_1+(Mk/\beta^2)]$$

$$K_1=\frac{\omega c}{2U_c},\quad k=\frac{\omega c}{2U},\quad \mu=\frac{Mk}{\beta^2},\quad \beta^2=1-M^2$$

and c is the blade chord. The sum of Eq. (2) and Eq. (3) becomes zero at the trailing edge, i.e., the Kutta condition is satisfied. In order to simplify the situation, it is assumed that a given frequency ω is associated with a single value of U_c, and thus K_1.

The complete surface pressure distribution on the $x_3=0$ plane can be obtained by adding the incident convecting surface pressure and the induced one. Then, as described earlier, the strength of the assumed dipole on the blade can be estimated by integrating the surface pressure jump. Following a suggestion of Amiet, the total load on the blade can be obtained as

$$L_T=\frac{bcP_0(K_1)e^{i2K_1}}{-2iK_1}\left[(1+i)\left\{\frac{\sqrt{(1+M)\mu+K_1}}{(1+M)\mu}\right.\right.$$
$$\left.\left.\times E^*[2\mu(1+M)]e^{-i2K_1}-E^*[2(\mu+\mu M+K_1)]\right\}+1\right] \tag{4}$$

where b is the span and $E^*(x)$ a combination of Fresnel integrals as

$$E^*(x)=\int_0^x(2\pi\xi)^{-1/2}e^{-i\xi}\mathrm{d}\xi$$

We assume that $L_T(w)$ acts at one point as an effective radius which models a blade. Then the required power spectral density of a point dipole can be calculated without any difficulty.

Application to a Rotor

In order to calculate the far-field acoustic intensity spectrum from a rotor, we first need the power spectral density of the blade loading. The blade load is assumed to act in a direction perpendicular to the rotor plane. Thus, the blade loading component in the direction of observation point can be represented as

$$L_r(\omega)=L_T(\omega)\sin\phi \tag{5}$$

where ϕ is the elevation angle of observation point from the rotor plane. Since $L_r(\omega)$ is a statistical quantity, we can obtain the power spectral density of blade loading as

$$D_r(\omega)=\lim_{T\to\infty}\left\{\frac{\pi}{T}L_r(\omega)L_r^*(\omega)\right\} \tag{6}$$

Thus, we obtain

$$D_r(\omega)=\frac{b^2c^2\sin^2\phi}{4}\lim_{T\to\infty}\left\{\frac{\pi}{T}P_0(\omega)P_0^*(\omega)\right\}$$

$$\times\frac{e^{i2K_1}e^{-i2K_1}}{-iK_1iK_1}(F+iG)(F-iG) \tag{7}$$

where

$$F=\left(\frac{\mu+M\mu+K_1}{\mu+M\mu}\right)^{1/2}\{(c_1+s_1)\cos2K_1$$
$$+(c_1-s_1)\sin2K_1\}+1-(c_2+s_2)$$

$$G=\left(\frac{\mu+M\mu+K_1}{\mu+M\mu}\right)^{1/2}\{(c_1-s_1)\cos 2K_1$$

$$-(c_1+s_1)\sin 2K_1\}-(c_2-s_2)$$

$$c_1-is_1=E^*[2\mu(1+M)]$$

$$c_2-is_2=E^*[2(\mu+\mu M+K_1)]$$

If we define the incident surface pressure power spectral density as

$$S_{pp}(\omega)=\lim_{T\to\infty}\left\{\frac{\pi}{T}P_0(\omega)P_0^*(\omega)\right\} \tag{8}$$

then we obtain the final expression of the power spectral density of the loads acting on a single blade as

$$D_r(\omega)=\frac{b^2c^2\sin^2\phi}{4K_1^2}(F^2+G^2)S_{pp}(\omega) \tag{9}$$

As mentioned earlier, we assume that blade-to-blade correlation can be neglected. Thus, we can add the intensity from each blade algebraically. Thus, Eq. (9) is substituted into Eq. (1) after it is multiplied by 8π to account for 1) a boundary layer on both upper and lower surfaces, 2) to convert to a single-sided ($0<f<\infty$) spectrum, and 3) to convert to a 1 Hz bandwidth. The result for a B-bladed rotor is given as

$$\langle S_1(x,f)\rangle=\frac{Bf^2b^2U_c^2\sin^2\phi}{2\pi\rho c_0^3r^2}\sum_{n=-\infty}^{n=\infty}\times\frac{F_g(|f-n\Omega|)S_{pp}(|f-n\Omega|)}{(f-n\Omega)^2}J_n^2\left(\frac{f}{\Omega}\alpha\right) \tag{10}$$

where $F_g=F^2+G^2$. In order to apply this equation to the trailing-edge noise prediction, we next present the appropriate spanwise loading corrections as discussed previously.

The acoustic analysis so far has concentrated on the net total load on each blade at a single point. This does not allow for interference effects between signals from separate portions of the same blade. Interference between signals at the observer's position arises because of the difference in phase of the signals which arrive there simultaneously. These phase differences are a result of variations in phase within the source distribution itself, as well as differences in the path length from source point to observer. Since a point dipole model by definition neglects both these effects, the total signal from each blade always arrives in-phase, and hence an overestimate of the acoustic radiation will result. Therefore, we need a correction factor to make the result realistic. A spanwise correlation length $\ell_2(\omega)$ of surface pressure can be defined as

$$\ell_2(\omega)=\frac{1}{S_{pp}(K_1,0)}\int_0^\infty S_{pp}(K_1,x_2)\,\mathrm{d}x_2 \tag{11}$$

We assume that each such strip radiates independently of its neighbors with random phase correlation between strips. This means that their respective acoustic intensities may be added directly. The radiated intensity will then be proportional to the square of the area $c^2\ell_2^2$ and the total from b/ℓ_2 such regions along the span will scale as $c^2b\ell_2$. For a point dipole, where $\ell_2/b\gg1$, the appropriate scaling factor is $(cb)^2$. Hence, we see that the radiation for finite correlation length is reduced by a factor ℓ_2/b. Thus, the spanwise correlation correction factor to the point dipole acoustic power spectral density will be

$$\left(1+\frac{b}{\ell_2(\omega)}\right)^{-1} \tag{12}$$

As discussed earlier, the chordwise distribution of loads can also influence the final results. However, as we are mainly interested in the results at observer angle not too close to the rotor plane, we will not include any corrections for chordwise load distribution.

The results obtained in the present work can be applied to any type of turbulent flow past a trailing edge so long as the assumption of stationary turbulence is reasonably well satisfied. At present, however, the only case for which surface pressure characteristics are sufficiently well documented is that of a turbulent boundary layer. Thus, we adopt the empirical expressions for the surface pressure spectrum as

$$S_{pp}(\bar\omega,0)=(\rho U^2/2)^2(\delta^*/U)S_0(\bar\omega) \tag{13a}$$

where $\bar\omega=\omega\delta^*/U$ and for a flat plate boundary layer[16]

$$S_0(\bar\omega)=2\times10^{-5}/(1+\bar\omega+0.217\bar\omega^2+0.00562\bar\omega^4) \tag{13b}$$

for $\bar\omega<20$. For an airfoil $S_0(\bar\omega)$ was fit to the data of Yu and Joshi[23] and the data of Brooks and Hodgson[24] giving the results

$$S_0(\bar\omega)=1.732\times10^{-3}\bar\omega/(1-5.489\bar\omega+36.74\bar\omega^2+0.1505\bar\omega^5) \tag{13c}$$

for $\bar\omega<0.06$ and

$$S_0(\bar\omega)=1.4216\times10^{-3}\bar\omega/(0.3261+4.1837\bar\omega+22.818\bar\omega^2+0.0013\bar\omega^3+0.0028\bar\omega^5) \tag{13d}$$

for $0.06<\bar\omega<20$. This curve ranges about 3 to 10 dB above the curve fit by Schlinker and Amiet[25] to the data of Yu and Joski.[23] The flat plate turbulent boundary layer displacement thickness δ^* at the trailing edge which is used in the definition of $\bar\omega$ is given by

$$\delta^*/c\approx0.047Re_c^{-1/5} \tag{14}$$

where Re_c is the Reynolds number based on chord c. The spanwise correlation length is given as

$$\ell_2(\bar\omega)=2.1(U_c/\omega) \tag{15}$$

where $U_c=0.8U$. Now substitute Eq. (13) into Eq. (10) and multiply by Eq. (12). We obtain the final result of trailing-edge noise radiation due to turbulent boundary layer on a hovering rotor as

$$\langle S_1(x,f)\rangle=\frac{bf^2b^2U_c^2\sin^2\phi}{2\pi\rho c_0^3r^2}\sum_{n=-\infty}^{n=\infty}\times\frac{F_g(|f-n\Omega|)S_{pp}(|f-n\Omega|)}{(f-n\Omega)^2\left(1+\dfrac{b}{\ell_2(|f-n\Omega|)}\right)}J_n^2\left(\frac{f}{\Omega}\alpha\right) \tag{16}$$

where $F_g=F^2+G^2$ and F and G are given in Eq. (7). We also have

$$\ell_2(|f-n\Omega|)\cong2.1\frac{U_c}{2\pi\epsilon|f-n\Omega|}$$

$$S_{pp}(|f-n\Omega|)=\left(\frac{1}{2}\rho U^2\right)^2\left(\frac{\delta^*}{U}\right)S_0(\bar\omega)$$

and

$$\bar\omega\equiv\frac{2\pi|f-n\Omega|\delta^*}{U}$$

Numerical Results and Discussion

In order to numerically evaluate Eq. (16), we must be able to truncate the infinite sum with negligible error. The basic idea of limiting the sum to a finite number of terms is due to the fact that $J_n(x)$ becomes small for $n \gg |x|$. By comparing results with different n's we determine that the series could be truncated with less than 1% error if n ranged between

$$n_{\min},\quad n_{\max} = +1.2M_0\,|\cos\phi|f/\Omega \tag{17}$$

The Bessel functions were determined by a backward recurrence relation which calculates $J_{n-1}(x)$ from $J_n(x)$. Fresnel integrals were evaluated by the subroutine given in the IBM computer manual based on the algorithm by Boersma.[26]

At present, there is no completely satisfactory data known to the authors with which we can compare our theory. The surface pressure spectrum due to the convecting turbulent flows is provided by experiments or empirical formulas. In addition to this, the influence of extraneous sources of noise not included in the theory should be minimized or subtracted. Also, the experiment should be carried out under clean inflow conditions so that the interaction between incident turbulence and boundary layer can be minimized. As no such data exists, we compare the theory to two existing helicopter rotor measurements.[27,28] The surface pressure spectrum was estimated by the empirical formula for a flat plate surface or an airfoil. There are no arbitrary or empirical factors in our analysis and, thus, the present theoretical calculations predict absolute rather than relative levels.

Comparison to Atmospheric Turbulence Inflow Noise

Theoretical calculations of rotor trailing-edge noise are shown in Figs. 5-7. The corresponding input parameters are presented in the figures along with the experimental results. First, we compare the calculated trailing-edge noise with the calculated atmospheric inflow turbulence noise for the two different rotors. The theoretical prediction of inflow turbulence noise was given by George and Kim.[29] It should be noted that the present theory of trailing-edge noise does not include the effects of inflow turbulence in the model. Thus, it predicts the ultimate noise level due to turbulent boundary layer when the influence of inflow turbulence does not exist. The rotor parameters are known in each case. However, the inflow turbulence values had to be estimated based on Refs. 30 and 31.

As can be seen from Fig. 5, for the full scale helicopter case, the trailing-edge noise due to convecting turbulent boundary layer becomes more important than the estimated typical daytime incident turbulence case for a relatively wide range of high frequencies under the given operating conditions. However, Fig. 6, for the inverted rotor near the ground, indicates that trailing-edge noise is well below the estimated typical daytime incident turbulence level for all except very high acoustic frequencies which are generally not of practical importance. In order to evaluate the effect of observer angle on the relative importance of trailing-edge noise, we calculate the sound level for an angle closer to the rotating axis for the same rotor as in Fig. 6. As can be seen from Fig. 7, the result is quite similar to that in Fig. 6.

As shown in Figs. 5-7, trailing edge noise is more important for the case of Johnson and Katz[27] than that of Leverton.[28] The most distinct difference between the two sets of data lies in the estimated values of the integral length scale of inflow turbulence. The length scale of Leverton's case was estimated as $\Lambda = 0.57$ m, while that of the Johnson and Katz case was estimated as $\Lambda = 27.5$ m. The smaller length scale inflow turbulence can increase the noise level at high frequencies as discussed by Homicz and George.[2] Thus, we can expect that moderate strength incident turbulence with an integral length scale comparable to that of boundary-layer turbulence can generate enough noise to mask the trailing-edge noise. However, if the length scale of weak inflow turbulence is much larger than that of boundary-layer turbulence, trailing-edge noise can become important for a relatively large range of high frequencies. The incident turbulence results presented in the figures hold only for conditions of moderate daytime turbulence intensity. As inflow turbulence intensity affects the

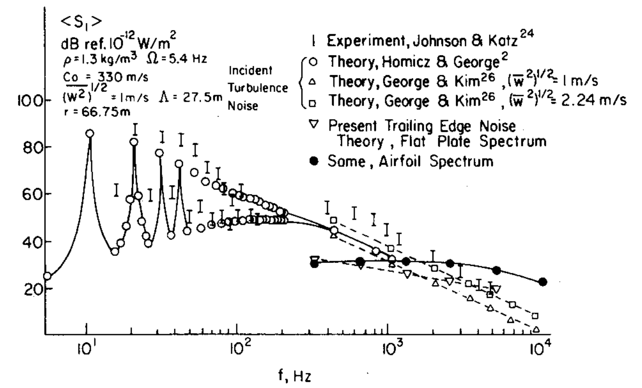

Fig. 5 Comparison of trailing-edge noise due to turbulent boundary layer with measurements and incident turbulence noise for a full-scale helicopter: $B = 2$, $M = 0.55$, $b = 6.3$ m, and $c = 0.530$ m.

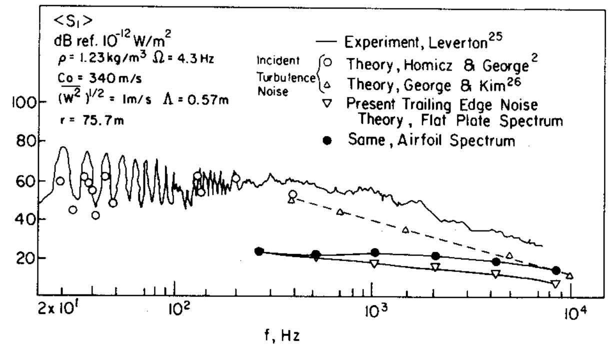

Fig. 6 Comparison of trailing-edge noise due to turbulent boundary layer with measurements and incident turbulence noise for the inverted full-scale rotor near the rotor plane; $\phi = -11.5$ deg, $B = 2$, $M = 0.51$, $b = 8.5$ m, $c = 0.430$ m.

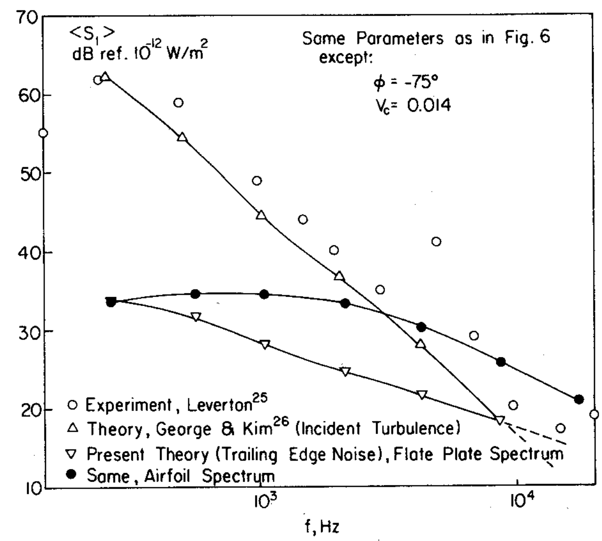

Fig. 7 Comparison of trailing-edge noise due to turbulent boundary layer with measurements and incident turbulence noise for the inverted full-scale rotor near the rotor axis; $\phi = -75$ deg.

inflow turbulence sound spectra over the entire frequency range, the relative importance of the trailing-edge noise will be affected as inflow turbulence changes (see Fig. 5). In a situation of strong inflow turbulence, if it is assumed that the two noise mechanisms can be considered independently, the inflow turbulence noise could mask the trailing-edge noise completely. However, high incident turbulence could also modify boundary-layer development, and hence, alter trailing-edge noise. Due to these considerations, some uncertainty exists in comparing the two mechanisms at high intensities.

Comparison of incident turbulence noise with trailing-edge noise indicates that both mechanisms can be important sources of broadband noise in hover depending on the operating environmental conditions. The relative importance of these two mechanisms appears to be related to the relative intensity and length scales between inflow and boundary-layer turbulence. Thus, under typical atmospheric conditions, the incident atmospheric turbulence noise may be less important than boundary-layer turbulence noise in stable air, at high altitudes, or during the night because of large integral length scales and low intensity of atmospheric turbulence. On the other hand, the atmospheric turbulence noise mechanism can dominate during daytime flight at low altitudes.

The recent work of George et al.[11] has shown that trailing-edge noise due to the turbulence in the forming tip vortex can also contribute to high-frequency broadband noise. This source is dependent on tip vortex strength and thus is particularly important for highly loaded blades. This mechanism, like boundary-layer turbulence noise, is relatively more important when incident turbulence is weak.

Comparison to Experiments

Next we evaluate the present theory by comparing with the existing experiments. As shown in Figs. 5-7, theoretically predicted trailing-edge noise spectra, particularly those based on a flat plate pressure spectrum, are below the experiments at moderate frequencies indicating that in these experiments other broadband sources seem to be more important. This probably resulted from the fact that experimental data were obtained under the influence of inflow turbulence which undoubtedly increased the noise level compared to "clean inflow" conditions. Possible underprediction may have come from Amiet's model of trailing-edge noise which we adopted in the present work and the simplifications imposed for the application to a rotor. To illustrate this, we will consider trailing-edge noise from a stationary airfoil. As shown in Fig. 2, the spectra level predicted by Amiet's model of trailing-edge noise due to turbulent boundary layer is somewhat below the experimental data. The disagreement between his theory and measured data could be attributed to the imposition of use of a flat plate pressure spectrum or to the Kutta condition at the trailing edge. As discussed by Ffowcs Williams,[32] predictions by theories with Kutta condition imposed should be regarded as lower-bound estimates on edge noise fields, while the simple scattering theories provide upper-bound estimates. According to Howe,[12] the difference between these two estimates can amount to 10 dB or more. This critical dependence on the Kutta condition can apparently result in uncertainty in the predicted noise levels. Well-controlled experiments are badly needed to clarify the precise conditions prevailing at the trailing edge. On the other hand, as we saw in Fig. 2, Amiet's theory appears to agree better with the case of a thinner airfoil at zero angle of attack for spectra's rate with frequency, while the discrepancy increases substantially at low frequencies for the case of a thicker airfoil or at nonzero angle of attack. This could be attributed to the fact that Amiet's model used the surface pressure spectrum of a flat plate at zero angle of attack while the airfoil pressure spectrum is larger in magnitude, particularly as the angle of attack or thickness increases. Since the turbulent eddy size scales with displacement thickness, large eddies may exist near the trailing edge, resulting in a boundary-layer pressure fluctuation spectrum dominated by lower frequencies. Therefore, the acoustic radiation from the actual blade would contain more energy at lower frequencies explaining some possible discrepancy between Amiet's theory and experiment.

For the observation angle close to the rotor plane, some underprediction may arise because of our approximation to the directivity of trailing-edge noise. Near the rotor axis, Fink's[15,33] prediction of rotor trailing-edge noise due to turbulent boundary layer matches the experimental data of Leverton[28] closely for the case in Fig. 7. In contrast, the predicted spectra level of incident atmospheric turbulence noise by George and Kim[29] also agrees well with the same experimental data. Considering that the incident turbulence could only have increased the experimental spectral level compared to that under an ideal clean inflow condition, Fink's formula may overpredict the trailing-edge noise as was shown for the case of a stationary airfoil in Fig. 2. Furthermore, recent experiments[34] demonstrate that under clean inflow conditions, the broadband spectrum from a rotor is relatively flat and shows little decay with increasing frequency. As Leverton's data show an apparent strong decay with increasing frequencies, this suggests that the noise was produced by incident turbulence. The spectrum of trailing-edge noise predicted by the present work shows the same trend as observed in the clean inflow experiments.[34] To theoretically predict rotor trailing-edge noise more accurately, we need more thorough investigations of the fundamental unresolved problems of trailing-edge noise which exist even for the case of nonrotating airfoils.

Conclusions

A method has been developed to analyze the rotor broadband noise due to the interaction of convecting turbulent eddies with the trailing edges of the blades. The present method modeled the trailing-edge noise from each blade as point dipole noise which was shown to be a good approximation to edge noise for angles not close to the rotor plane. The analysis was applied to the trailing-edge noise due to a turbulent boundary layer on a flat plate or airfoil blade. Trailing-edge noise is compared with incident atmospheric turbulence noise. The relative importance of these two mechanisms varies with frequency and is related to the different magnitudes of intensity and length scales between inflow and boundary-layer turbulence. Trailing-edge noise can be dominant at high frequencies when inflow turbulence is weak as in night time or stable air flight conditions. The calculated results tend to fall below the available experimental data which include strong incident turbulence noise. The present analysis assumes the Kutta condition at the trailing edge and thus gives a lower bound on the radiated sound. Further improvement of the theory may be necessary to include a modified Kutta condition. The present method may be extended to predict the noise level from other turbulent sources such as tip flows and separated flows, which also seem to be involved in broadband rotor noise.

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